Power in an ac Circuit

For a dc circuit the power is related to voltage and current as follows:

$$P = V \cdot I$$

In an ac circuit voltage and current depend on time:

$$V \longrightarrow v(t) = V_o \cdot \sin(\omega t)$$

$$I \longrightarrow i(t) = I_o \cdot \sin(\omega t - \Delta\varphi)$$

The instantaneous value of the power at time t is therefore:

$$p(t) = v(t) \cdot i(t) = V_o \cdot \sin(\omega t) \cdot I_o \cdot \sin(\omega t - \Delta\varphi).$$

Using the trigonometric relation

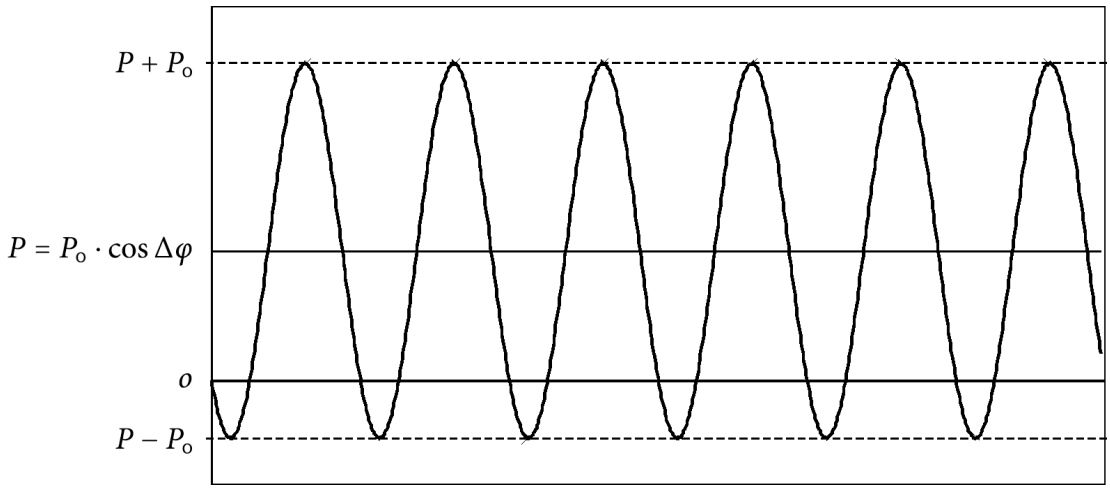
$$\sin\alpha \cdot \sin\beta = \frac{1}{2}\left(\cos(\alpha - \beta) - \cos(\alpha + \beta)\right),$$

and the substitutions $\alpha = \omega t$ and $\beta = \omega t - \Delta\varphi$ we get

$$p(t) = V_o \cdot I_o \cdot \frac{1}{2}\left(\cos(\Delta\varphi) - \cos(2\omega t - \Delta\varphi)\right) = P_o \cdot \left(\cos\Delta\varphi - \cos(2\omega t - \Delta\varphi)\right),$$

where $P_o = \frac{1}{2} V_o \cdot I_o$.

We thus find that the power oscillates with amplitude P_0 and twice the frequency of the applied voltage around the average value $P = P_o \cdot \Delta\varphi$ (see figure).



In most practical applications only the average value of the power is of any importance. From the figure it is obvious that the average power is

$$P = \overline{p(t)} = P_o \cdot \cos\Delta\varphi = \frac{1}{2} V_o \cdot I_o \cdot \cos\Delta\varphi$$

Using the *root mean squares (rms values)* or *effective values* $V_{rms} = \frac{V_o}{\sqrt{2}}$ and $I_{rms} = \frac{I_o}{\sqrt{2}}$ for the voltage and the current we end up with an expression very similar to the one describing the power in a dc circuit:

In an ac circuit with effective voltage V_{rms} and effective current I_{rms}, the *effective power* is

$$P = V_{rms} \cdot I_{rms} \cdot \cos\Delta\varphi,$$

where the product $S = V_{rms} \cdot I_{rms}$ is the *apparent power* and $\cos\Delta\varphi$ the *power factor*.

In the data sheet of ac appliances we usually find the rms values for voltage and current and the effective power or the power factor.